GRAND TEST - 1 (FIRST BOOK)

MM: 100 TIME: 3 Hrs

QUESTIONS CARRYING ONE MARK EACH

1. Write the value of
$$\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right].$$

2. What is the principal value of
$$\cos^{-1}\left(\cos\frac{2\pi}{3}\right)+\sin^{-1}\left(\sin\frac{2\pi}{3}\right).$$

3. Find the value of $y-x$ from following equation
$$2\begin{bmatrix}x & 5\\ 7 & y-3\end{bmatrix}+\begin{bmatrix}3 & -4\\ 1 & 2\end{bmatrix}=\begin{bmatrix}7 & 6\\ 15 & 14\end{bmatrix}.$$

4. If $A^T=\begin{bmatrix}3 & 4\\ -1 & 2\\ 0 & 1\end{bmatrix}$ and $B=\begin{bmatrix}-1 & 2 & 1\\ 1 & 2 & 3\end{bmatrix}$, then find A^T-B^T.

5. If A_{ij} is the cofactor of the element a_{ij} of the determinant $\begin{vmatrix}2 & -3 & 5\\ 6 & 0 & 4\\ 1 & 5 & -7\end{vmatrix}$, then write the value of $a_{32}\cdot A_{32}$.

6. What positive value of x makes following pair of determinants equal?
$$\begin{vmatrix}2x & 3\\ 5 & x\end{vmatrix}=\begin{vmatrix}16 & 3\\ 5 & 2\end{vmatrix}$$

QUESTIONS CARRYING FOUR MARKS EACH

7. Show that the relation S in set $A=\{x\in Z:0\le x\le 12\}$ given by $S=\{(a,b):a,b\in Z,|a-b|$ is divisible by $4\}$ is an equivalence relation. Find the set of all elements related to A.

8. Consider the binary operations $*:R\times R\to R$ and $o:R\times R\to R$ defined as $a*b=|a-b|$ and $aob=a$. For all $a,b\in R$. Show that $*$ is commutative but not associative, 'o' is associative but not commutative.

9. Express the following matrix as a sum of a symmetric and a skew-symmetric matrix and verify your result $\begin{bmatrix}3 & -2 & -4\\ 3 & -2 & -5\\ -1 & 1 & 2\end{bmatrix}$.

10. Prove that
$$\cos^{-1}\left(\frac{4}{5}\right)+\cos^{-1}\left(\frac{12}{13}\right)=\cos^{-1}\left(\frac{33}{65}\right).$$

11. Prove the following
$$\cot^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]=\frac{x}{2};\ x\in\left(0,\frac{\pi}{4}\right).$$

12. Show that $\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right)=\frac{4-\sqrt{7}}{3}$.

OR

Solve the following equation
$$\cos(\tan^{-1}x)=\sin\left(\cot^{-1}\frac{3}{4}\right).$$

13. If $A=\begin{bmatrix}2 & 0 & 1\\ 2 & 1 & 3\\ 1 & -1 & 0\end{bmatrix}$, then find value of $A^2-3A+2I$.

14. Using properties of determinants, prove the following
$$\begin{vmatrix}3x & -x+y & -x+z\\ x-y & 3y & z-y\\ x-z & y-z & 3z\end{vmatrix}$$
$$=3(x+y+z)(xy+yz+zx).$$

15. Find the value of k, for which
$$f(x)=\begin{cases}\dfrac{\sqrt{1+kx}-\sqrt{1-kx}}{x}, & \text{if } -1\le x<0\\ \dfrac{2x+1}{x-1}, & \text{if } 0\le x<1\end{cases}$$ is continuous at $x=0$.

16. If the radius of sphere is measured as 9 cm with an error of 0.03 cm, then find the approximate error in calculating its surface area.

Find the intervals in which the function $f(x)=\sin x+\cos x,\ 0\le x\le 2\pi$ is strictly increasing or strictly decreasing.

17. If $x^y=e^{x-y}$, then prove that $\frac{dy}{dx}=\frac{\log x}{(1+\log x)^2}$.

OR

Differentiate $\tan^{-1}\left[\frac{\sqrt{1+x^2}-1}{x}\right]$ w.r.t. x.

18. If $x=a\cos^3\theta$ and $y=a\sin^3\theta$, then find the value of $\frac{d^2y}{dx^2}$ at $\theta=\frac{\pi}{6}$.

If $x\sin(a+y)+\sin a\cos(a+y)=0$, then prove that $\frac{dy}{dx}=\frac{\sin^2(a+y)}{\sin a}$.

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19. If $f(x)$ defined by the following, is continuous at $x = 0$, then find the values of a, b and c.

$$f(x) = \begin{cases} \dfrac{\sin(a+1)x + \sin x}{x}, & \text{if } x < 0 \\ c, & \text{if } x = 0 \\ \dfrac{\sqrt{x+bx^2} - \sqrt{x}}{bx^{3/2}}, & \text{if } x > 0 \end{cases}$$

QUESTIONS CARRYING SIX MARKS EACH

20. If the length of three sides of a trapezium other than the base are each equal to 10 cm, then find the area of the trapezium, when it is maximum.

21. Using elementary row transformation find the inverse of

$$\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

OR

Using elementary row transformation find the inverse of

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$$

22. Using matrices, solve the follcwing system of equations

$$x + 2y - 3z = -4$$
$$2x + 3y + 2z = 2$$
$$\text{and} \quad 3x - 3y - 4z = 11$$

23. If $y = (x)^x + (\sin x)^x$, then find $\dfrac{dy}{dx}$.

24. Find the intervals in which the following function $f(x) = 20 - 9x + 6x^2 - x^3$ is

(i) strictly increasing.

(ii) strictly decreasing.

OR

Find the equations of tangents to the curve $3x^2 - y^2 = 8$, which passes through the point $\left(\dfrac{4}{3}, 0\right)$.

25. If the function $f : R \to R$ is given by $f(x) = \dfrac{x+3}{3}$ and $g : R \to R$ is given by $g(x) = 2x - 3$, then find (i) fog and (ii) gof. Is $f^{-1} = g$?

26. Find the point on the curve $y^2 = 2x$ which is at a minimum distance from the point $(1, 4)$.

ALL THE BEST !!